1. You are going to play 2 games of chess with an opponent whom you have never played against before (for the sake of this problem). Your opponent is equally likely to be a beginner, intermediate, or a master. Depending on   
   (a) What is your probability of winning the first game?

Let Wi be the event of winning the ith game. By the law of total probability,

P(W1) = (0.9 + 0.5 + 0.3)/3 = 17/30.

(b) Congratulations: you won the first game! Given this information, what is the probability that you will also win the second game

We have P(W2|W1) = P(W2,W1)/P(W1). The denominator is known from

(a), while the numerator can be found by conditioning on the skill level of the

opponent:

P(W1,W2) = ( 1 / 3) P(W1,W2|beginner) + ( 1 / 3) P(W1,W2|intermediate) + ( 1 / 3)

P(W1,W2|expert)

Since W1 and W2 are conditionally independent given the skill level of the

opponent, this becomes

P(W1,W2) = (0.92 + 0.52 + 0.32) / 3 = 23/60.

So P(W2|W1) = (23/60) / (17/30)

= 23/34.

(c) Explain the distinction between assuming that the outcomes of the games are independent and assuming that they are conditionally independent given the opponent’s skill level. Which of these assumptions seems more reasonable, and why?

Independence here means that knowing one game’s outcome gives no information

about the other game’s outcome, while conditional independence is

the same statement where all probabilities are conditional on the opponent’s

skill level. Conditional independence given the opponent’s skill level is a more reasonable assumption here. This is because winning the first game gives information

about the opponent’s skill level, which in turn gives information about

the result of the second game.

That is, if the opponent’s skill level is treated as fixed and known, then it may

be reasonable to assume independence of games given this information; with

the opponent’s skill level random, earlier games can be used to help infer the

opponent’s skill level, which a↵ects the probabilities for future games.